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CONDITIONS FOR SIMILITUDE IN THE CASE OF SLENDER BODIES WITH A HYPERSONIC TURBULENT BOUNDARY LAYER

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ABSTRACT

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The conditions for similitude in hypersonic flow past flat ($\nu = 0$) and axisymmetric ($\nu = 1$) slender blunt bodies with a turbulent boundary layer on the surface are investigated, assuming variable entropy at the outer edge of the layer. The case of a laminar boundary layer has been treated previously by the author (ref. 1) and without regard for the influence of variable entropy by Cheng, et al. (ref. 2). Author

In hypersonic flow past slender blunt bodies, the boundary layer on their lateral surface is developed inside an inviscid, intensely turbulent, high-entropy layer produced by the gas passing through a steeply inclined shock wave segment formed in the vicinity of the nose.

Due to the high temperature and low density of the gas, the pressure across the high-entropy layer may be assumed constant.

Strong transverse entropy gradients in the inviscid high-entropy layer cause a considerable variation in entropy over the edge of the boundary layer, which can have an appreciable effect on its characteristics,[1] particularly in the case of a turbulent boundary layer with an effective adiabatic exponent γ_0 near unity. This can be amply confirmed by the use, for example, of empirical

[1]This was first pointed out by Ferri and Libby (ref. 3) and later investigated by a number of authors.

Numbers in the margin indicate pagination in the original foreign text.

formulas (ref. 4) that have been verified in the range of Mach numbers $M_1 = 2$ to 10:

$$c_q = \frac{q_w}{\rho_\infty U^3} = 0.0296\, \sigma^{-2/3} R_*^{-0.2} \rho_* u_1 (i_r - i_w)$$
$$c_f = \frac{\tau_w}{\rho_\infty U^2} = 2\sigma^{2/3} u_1 c_q \tag{1}$$
$$\left(R_* = \frac{\rho_* u_1}{\mu_*} R_\infty,\ R_\infty = \frac{\rho_\infty l U}{\mu_\infty},\ i_r = i_1 + 0.45,\ i_* = 0.28\, i_1 + 0.22\, i_r + 0.5\, i_w\right)$$

Given constance σ, $\rho\mu$, and ρi, a Free-stream Mach number $M = \infty$, and $p \approx 0.03$, these equations yield values of c_f and c_q less for $S_1 = S_0$ than for $S_1 \ll S_0$ (i.e., for $M_1 \gg 1$ and $u \approx 1$) by a factor of 1.8 and 1.5 respectively at $\gamma_0 = 1.4$, and by a factor of 4 and 2.1 respectively at $\gamma_0 = 1.1$.

The following notation is used: τ_w and q_w are the local friction and heat flux, l is the length of the body, ρ_∞, μ_∞, and U are the density, viscosity, and velocity of the free stream, $\rho_\infty U^2 p$, iU^2, $\rho\rho_\infty$, $\mu\mu_\infty$, and uU are the pressure, enthalpy, density, viscosity, and velocity of the air, respectively, S is the entropy, σ the Prandtl number. The subscripts 1, w, and 0 refer to the values of the variables at the edge of the boundary layer, on the surface of the body, and at the blunt body critical point. The asterisk indicates the local pressure value of p and the so-called controlling enthalpy i_*.

Let us assume that the equation of state, viscosity, and Prandtl number /14 in the high-entropy layer are given by the equations

$$p = k\rho f_1(i, k),\quad \mu = c\mu_0 f_2(i, k),\quad \sigma = \sigma(i, k),\quad k = \frac{p_0}{\rho_0}$$
$$\left(c = \text{const} \sim 1,\ f_1 \sim \frac{i}{i_0}\right) \tag{2}$$

In the shock layer (high-density layer contiguous to the compression shock), the air is assumed to be an ideal gas with constant adiabatic exponent γ.

Let $x_1 = x\,l$ and $y_1 = y\,l$ be the coordinates along the axis and along the normal to the surface of a body whose shape is specified by the equation

$$r = \beta l r_w(x) + \frac{1}{2} d$$

where d and c_x are the midsection diameter and drag coefficient of the nose, α and β are characteristic slopes of the shock wave and surface of the body. Then, in order to have similitude of flow, including that in the high-entropy layer, in the case of inviscid flow past slender blunt bodies of identical configuration $r_w(x)$, it is required, according to references 1 and 5, that the following parameters be invariant in similar cases:

$$\theta = M\beta, \quad K = \frac{1}{2^{\nu}} c_x \beta^{-3-\nu} \left(\frac{d}{2l}\right)^{1+\nu}, \quad \gamma, \quad k \quad \text{for } d \ll \alpha l;\ \alpha, \beta \ll 1 \tag{3}$$

and that the following function be invariant in the high-entropy layer:

$$s = s(\varphi) \qquad \left(s = \sin^2 \varepsilon,\ \varphi = \frac{\psi}{c_x},\ \psi^{(1)} = \pi^{\nu} \left(\frac{1}{2} d\right)^{1+\nu} \rho_\infty U \psi\right) \tag{4}$$

where $\psi^{(1)}$ is the dimensional streaming function, ϵ is the slope of the shock wave at its points of intersection with streamline ψ.

Similitude will hold in the high-entropy layer under the condition that at constant entropy the dependence of i on p can be neglected, which is true when $\gamma_0 - 1 \ll 1$ or when the variations in α are small in similar cases.

With an identical equation of state, the quantities i(s, k) and p(s, k) immediately behind the shock wave will be the same,[2] hence in similar cases the functions i(ϕ) and u(ϕ) in the high-entropy layer will be the same.

In analyzing a turbulent boundary layer, we will assume that the quantities τ_w and q_w depend on the local boundary parameters as in the case of a boundary layer on a plate, and we will take into account their dependence on the distribution of the functions

$$i_1(x^1), \quad i_w(x^1), \quad p(x^1) \quad \text{for } x^1 < x \tag{5}$$

by means of the coefficients (F and G in eqs. (6)), which are functionals of the functions (5). Then, extending equations (2) to the entire region of the boundary layer, the existing empirical relations for plates (see, e.g., ref. 3) can be generalized to the case of the inconstant functions (5) as follows:

$$c_f = \beta^2 F(x)\, k^{-0.8} (P u_1)^{0.8} \Omega, \quad c_q = G(x)\, c_f, \quad \Omega = \left(\frac{c\mu_0}{R_\infty}\right)^{0.2} \beta^{-1.4}, \; P = \frac{p}{\beta^2} \tag{6}$$

The momentum and energy equations for the turbulent boundary layer are /15 used in integral form:

$$\frac{d}{dx}\int_0^\delta r^\nu \rho u\,(u_1 - u)\,dy = \frac{dp}{dx}\int_0^\delta r^\nu\,dy + \frac{du_1}{dx}\int_0^\delta r^\nu \rho u\,dy + (\beta r_w)^\nu c_f$$
$$\frac{d}{dx}\int_0^\delta \rho u r^\nu\,(i_0 - i_0')\,dy = (\beta r_w)^\nu c_q \qquad \left(i_0' = i + \frac{u^2}{2}\right) \tag{7}$$

[2] Assuming everywhere an ideal polytropic gas, the effect of the Mach number M on the profile of the shock wave in its leading portion and the relations describing it are accounted for in the variable k(γ, M). This effect of M will be neglected in the case of the general equation of state.

Regarding the profiles of the quantities u/u_1 and i/i_0, we will assume that they depend on the variable $\eta = y/\delta$ (where δ is the thickness of the boundary layer), on the local values of i_1 and i_w and derivative $\partial i/\partial\eta$ at $\eta = 1$, as well as on the two parameters[3] λ_1 and λ_2. Then the integrals

$$\delta^{**} = \int_0^1 \frac{\rho u}{\rho_0' u_1}\left(1 - \frac{u}{u_1}\right) g^\nu \, d\eta, \qquad \vartheta = \int_0^1 \frac{\rho u}{\rho_0' u_1}(i_0 - i_0') g^\nu \, d\eta$$
$$I = \int_0^1 \frac{\rho u}{\rho_0' u_1} g^\nu \, d\eta \qquad \left(g = 1 + \frac{\delta}{\beta r_w}\eta\right) \tag{8}$$

will also depend on these parameters and the boundary conditions, where ρ_∞ ρ_0' is the density at the given local pressure and stagnation enthalpy.

Making use of equations (6) and (8), equations (7) reduce to the form

$$\frac{d}{dx}(r_w^\nu P u_1^2 Y_1 \delta^{**}) = \frac{k}{2^\nu} r_w^{1+\nu} \frac{dP}{dx} Y_1 (g_1^{1+\nu} - 1) + \frac{du_1}{dx} P u_1 Y_1 I r_w^\nu + k^{0.2} r_w^\nu F\Omega$$
$$\frac{d}{dx}(r_w^\nu P u_1 Y_1 \vartheta) = k^{0.2} r_w^\nu G F\Omega \qquad \left(Y = \frac{y}{\beta}, Y_1 = \frac{\delta}{\beta}\right) \tag{9}$$

We note that, assuming known laws of friction and heat transfer (6), it may then be presumed conceptually possible to determine the coefficients F and G from the distribution of the functions (5). Specifically, these coefficients can be prescribed functions of the local boundary conditions and parameters δ^{**}, ϑ, λ_1, λ_2.

The quantities $i_1 = i(\phi_1)$ and $u_1 = u(\phi_1)$ are determined from the equations of mass flow through the cross section of the boundary layer:

[3] In a more general formulation of the problem we would replace the dependence of the profiles on λ_1 and λ_2 by their functional relation with the distribution of the functions (5). For $\nu = 1$, these profiles and the coefficients F and G may also depend on the parameter $\delta/\beta r_w$.

$$\varphi_1(x) = 2^{\nu-1}\frac{u_1 r_w^{\nu}}{f_1(i_1)}\frac{P}{kK}Y_1 I \tag{10}$$

The relative thickness of the boundary layer Y_1 must be determined, as in the laminar case, from the stipulation that the velocity and enthalpy profiles join smoothly between the inviscid high-entropy layer and boundary layer:

$$\begin{gathered}\frac{\partial i}{\partial \eta} = Y_1\frac{\partial i}{\partial Y} = 2^{\nu+1}(r_w + Y_1)^{\nu} Y_1 \frac{Pu_1}{kf_1(i_1)K}\frac{di_1}{d\varphi} \\ u_1\frac{\partial u_1}{\partial \eta} = -\frac{\partial i}{\partial \eta}, \qquad i = i_1, \quad u = u_1 \quad \text{for } \eta = 1\end{gathered} \tag{11}$$

If the boundary layer exceeds the limits of the high-entropy layer, the conditions (11) then assume the form

$$\frac{\partial u}{\partial \eta} \sim \frac{\partial i}{\partial \eta} \sim Y_1\alpha^2 \approx 0, \qquad i_1 \approx \alpha^2 \approx 0, \qquad u_1 \approx 1 \quad \text{for } \eta = 1 \tag{12}$$

The term containing du_1/dx drops out of equations (9), and equation (10)/16 becomes superfluous. It is expected that the characteristics of the turbulent boundary layer (the functions F and G in particular), as in the laminar case, will not depend on i_1 for $i_1 \ll 1$, i.e., on M_1 for $M_1 \gg 1$. This is corroborated to a certain extent by the empirical formula (1), which does not depend on i_1 as $i_1 \rightarrow 0$.

Equations (9) to (11) comprise a closed system in the unknowns λ_1, λ_2, ϕ_1, and Y_1. In order to solve equations (10) we need to know the values of λ_1 and λ_2 near the nose ($x \approx 0$), which must be ascertained in terms of the quantities $\delta\delta^{**}$ and $\delta\delta$ in the region where the nose joins the lateral surface. Since $p \sim \alpha^2$, the ratio of the total heat flux or friction forces on the nose to their values on the lateral surface, as well as the corresponding ratios for $\delta\vartheta$ and $\delta\delta^{**}$, according to (1), are of the order[4].

$$\omega \sim \left(\frac{d}{l}\right)^{0.8} \alpha^{-1.6} \left(1 + \frac{\beta l}{d}\right)^{-\nu} \tag{A}$$

As in reference 1, the category of situations investigated will be delimited by the condition[5].

$$K \lessapprox \alpha^{-m} \left(\frac{\alpha}{\beta}\right)^{3+\nu} \quad (0 \leqslant m < 1) \qquad \alpha^{j} \sim \beta \gtrapprox \frac{d}{l} \quad \text{for } \nu = 1 \tag{13}$$

Then

$$\omega \lessapprox \alpha^{m_1} \ll 1 \quad \text{for } 1 \leqslant j < 2 - \frac{9m}{10} \quad \left(m_1 = \frac{(4+5\nu)(3+\nu-m) - 8(1+\nu)}{5(1+\nu)} - \nu_j\right) \tag{B}$$

Consequently, when the conditions (13) are satisfied, we have $\delta(0)/\delta(1) \ll 1$. Hence, it may be assumed that $\delta(0) \approx 0$ and the effect of the conditions at $x \approx 0$ on the solution of equations (10) can be neglected.

[4]This presupposes the existence throughout of a turbulent boundary layer. Should a laminar layer prevail on the blunt body, this will only (for sufficiently large R_∞) reduce the value of ω.

[5]The maximum order of magnitude of $K_\alpha = K(\beta/\alpha)^{3+\nu}$ will occur when the order of the perturbations injected into the airstream by the body are governed by the influence of the nose. Inasmuch as in this case $K_\alpha \sim 1$, the first condition (13) will almost always be satisfied for $m = 0$. The condition $\omega \ll 1$ cannot be met for a blunt cylinder, because now it is possible for $\omega \sim (d/l\ \alpha^2)^{4/5} \sim 1$; for a plate, $\omega \sim (d/l)^{4/15} \ll 1$.

The influence of the boundary layer on the outer flow can be taken into account by increasing the thickness of the layer to the value δ^* at which the boundary layer is exceeded, and replacing the shape of the body by the effective shape

$$r = \beta\,(r_w(x) + Y^*) \qquad \left(Y^* = \frac{\delta^*}{\beta}\right) \tag{14}$$

If $1\delta_2 = \beta 1 Y_2$ is the thickness of that portion of the inviscid high-entropy layer bounded by the streamline ψ_1 at the local pressure, then clearly

$$Y^* = Y_1 - Y_2 \qquad (r_w + Y_2)^{1+\nu} = r_w^{1+\nu} + \frac{kK}{P}\int_0^{\varphi_1} \frac{f_1(i_2)\,d\varphi}{u_2} \tag{15}$$

Here the subscript 2 refers to the values of the quantities in the inviscid high-entropy layer.

Equations (9) to (15) contain, in addition to the similarity criteria (3) and (4), the parameter Ω and the function $i_w(x)$. From this we are able to formulate the following law of similitude.

In hypersonic flow past slender blunt bodies with a turbulent boundary layer on their lateral surface, similarity of flow will hold true for identical values of the parameters and functions /17

$$\theta, \quad k, \quad K, \quad \Omega, \quad r_w(x), \quad s(\varphi), \quad i_w(x) \tag{C}$$

Here the functions $P(x, Y)$, $V(x, Y) = v/\beta$, $\rho(x, Y)$ in the shock layer and $P(x, Y)$, $u(x, Y)$, $i(x, Y)$ in the inviscid high-entropy and boundary layers will be identical in similar cases.

Furthermore, the distribution of the quantities P, c_f/β^3, and c_q/β^3 will be the same on the surface of the body.

For bodies with a power-law configuration $r_w = x^n$, the characteristic length l will be absent (refs. 4 and 5) and may be eliminated from further consideration, letting, for example, $K = 2^{-\nu}$. The variables x, Y and the parameter Ω in this case must be replaced by the respective quantities

$$\xi = \frac{2x_1}{d} c_x^{-\frac{1}{\nu+1}} \beta^{\frac{3+\nu}{1+\nu}}, \qquad \zeta = \frac{2y_1}{d} c_x^{-\frac{1}{1+\nu}} \beta^{\frac{2}{1+\nu}}$$
$$\Omega_0 = \left(\frac{2\mu_0 c}{R_d}\right)^{0.2} c_x^{-\frac{0.2}{1+\nu}} \beta^{-\frac{0.8+1.2\nu}{1+\nu}}, \qquad R_d = R_\infty \frac{d}{2l} \tag{16}$$

In the case of a laminar boundary layer, the parameters analogous to Ω and Ω_0 have the form

$$\Omega_l = \frac{\chi}{\theta^2} = \left(\frac{\mu_0 c}{R_l}\right)^{1/2} \frac{1}{\beta^2}, \qquad \Omega_{0l} = \left(\frac{2\mu_0 c}{R_d}\right)^{1/2} c_x^{-\frac{1+\nu}{2}} \beta^{-\frac{2(1+\nu)}{1+3\nu}} \tag{E}$$

Here χ is the usual interaction parameter of the laminar boundary layer with the outer flow (ref. 1).

The quantities (16) can be replaced by their equivalents for $\beta \longrightarrow 0$:

$$\xi_1 = \xi \Omega_0^{\frac{3+\nu}{0.8+1.2\nu}}, \qquad \zeta_1 = \zeta \Omega_0^{\frac{2}{0.8+1.2\nu}}, \qquad \Omega_M = \Omega_0 \theta^{\frac{0.8+1.2\nu}{1+\nu}} \tag{F}$$

Now in similar cases we find identical distribution of the quantities

$$p(\beta\Omega_0^{\varkappa})^{-2}, \qquad c_f(\beta\Omega_0^{\varkappa})^{-3}, \qquad c_q(\beta\Omega_0^{\varkappa})^{-3} \qquad \left(\varkappa = \frac{1+\nu}{0.8+1.2\nu}\right) \tag{G}$$

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